Rice University
Department of Economics
ECON 598 - Continuous Time Economics
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Instructors:

Paul Barton
Baker Hall 359
paul.j.barton@rice.edu

Hülya Eraslan
Baker Hall 268
eraslan@rice.edu

Overview

The goal of this "guided group independent study" course is to learn the tools used in continuous time economic models in general, and those used in game theory in particular, and to familiarize ourselves with the continuous-time game theory literature. To achieve these goals, this course will involve a combination of instruction and group work, with bi-weekly instruction supplemented by meeting twice per week for group work. The plan is to spend the majority of the semester on learning the mathematical tools needed to properly understand continuous-time economic models, followed by an introduction to continuous-time game theory. This course may extend past the end of the semester.

Registration

This course is considered an independent study. To register for this course one will need to bring a signed special registration form to the registrar.

Textbooks

1. Patrick Billingsley 1995, *Probability and Measure.* Wiley. ISBN 978-0-471-00710-4.
2. Ioannis Karatzas and Steven Shreve 2014, *Brownian Motion and Stochastic Calculus*. Springer. ISBN 978-1-4612-0949-2.

Prerequisites

While there are no official prerequisites, this course assumes that the student has a background in measure-theoretic probability equivalent to having taken STAT/CAAM 581 at Rice.

Guide For Preparation

What follows are some topics to look into in preparing for the course. The idea is to review concepts in mathematical probability. You will find that understanding these topics requires knowing about measures

and Lebesgue integration. A good grasp of those ideas will be essential, so what follows is meant to be in addition to abstract integration.

1. Dynkin's π-λ theorem, also called the Monotone Class Theorem. Study the definition of π and λ-systems, read and understand the proof the π-λ theorem. Find proofs of formal results which rely on this theorem, and study the application. Many applications are hand-wavy; e.g. "by a monotone class argument...". See if you can fill in this argument.
2. Note the definition of completeness, but don't worry yet about why it is necessary.
3. Study some theorems regarding independence.
4. Study the definition of events that occur *infinitely often* and *ultimately*, and study the Kolmogorov 0-1 law.
5. Study the Borel-Cantelli lemma.
6. Review convergence in probability, in distribution, and almost surely. Study the weak and strong laws of large numbers, and the central limit theorem.

Grading

Students will be assigned letter grades, which will be determined on the basis of class participation and in-class presentation of homework solutions.

Tentative List of Topics

1. Probability Theory
   (a) Conditional expectations
   (b) Markov chains
   (c) Martingales
2. Continuous Stochastic Processes
   (a) Brownian motion
   (b) Stochastic integration
   (c) Stochastic differential equations
3. Optimal Control
   (a) Dynamic Programming
   (b) Viscosity Solutions
4. A discussion of a few continuous-time papers chosen by the students. Possible choices (among many others) include:
   (a) Anderson, A. and Smith, L. 2013. Dynamic Deception. *American Economic Review*, 103(7): pp.2811-2847.
   (b) Bohren, A., 2014. Stochastic Games in Continuous Time: Persistent Actions in Long-Run Relationships. Working Paper
   (c) Daley, B. and Green, B., 2012. Waiting for News in the Market for Lemons. *Econometrica*, 80(4), pp.1433-1504.
   (d) Faingold, E. and Sannikov, Y., 2011. Reputation in Continuous-Time Games. *Econometrica*, 79(3), pp.773-876.

(e) Fudenberg, D. and Levine, D.K., 2007. Continuous time limits of repeated games with imperfect public monitoring. *Review of Economic Dynamics*, 10(2), pp.173-192.

(f) Holmstrom, B. and Milgrom, P., 1987. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica*, pp.303-328.

(g) Kolb, A.M., 2015. Optimal entry timing. *Journal of Economic Theory*, 157, pp.973-1000.

(h) Sannikov, Y., 2007. Games with imperfectly observable actions in continuous time. *Econometrica*, 75(5), pp.1285-1329.

(i) Sannikov, Y., 2008. A continuous-time version of the principal-agent problem. *The Review of Economic Studies*, 75(3), pp.957-984.